PULSE INPUT SIZING FOR CONSTRUCTING REDUCED ORDER MODELS OF THE EULER EQUATIONS

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Abstract: Sizing the pulse magnitude for constructing dynamically time linear reduced order models can become a labourious process of trial and error for the aerodynamicist. Improper sizing of the pulse may lead to poor convergence or breakdown of the flow equations. In this paper we present a method to size the pulse input using classical one dimensional piston theory.

1 INTRODUCTION

Dynamically linear reduced order models (ROM) constructed about a nonlinear mean flow solution have become a powerful tool in modern aeroelastics. Constructing a ROM requires the time history of the aerodynamic system to a general force input. A popular method of obtaining the time history is to calculate the linear response of a CFD scheme to a forced non periodic pulse [1–11] . An improperly sized pulse can cause non-physical solutions of the Euler equations from which the solution cannot recover. Without prior insight into the transient aerodynamic response, the task of choosing the initial input often results in a process of trial and error. Here we present a method using classical one dimensional piston theory to estimate the initial unsteady linear response to a forced input. The robust closed-form nature of the equations make them ideal for an automated process.

To correctly size a pulse input to the non-linear Euler equations, it is important to estimate the pulse response of the Euler scheme. Experience has shown that the maximum change in pressure usually occurs on the first time step of the CFD scheme. We will demonstrate that the instantaneous pulse response can be closely approximated by classical one dimensional piston theory. Here we present an example of a transonic 2D aerofoil undergoing heave, pitch and flap oscillations. The process however remains applicable in three dimensions.

2 APPLICATION TO REDUCED ORDER MODELING

The CFD code used is a modified Jameson cell centered finite volume scheme for the solution of the Euler equations on a moving mesh [12–14].

The Euler equations can be expressed as a nonlinear state space formulation:

$$\frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)) \quad (1)$$

$$\mathbf{y}(t) = \mathbf{h}(t, \mathbf{x}(t), \mathbf{u}(t)) \quad (2)$$

where for an aerofoil with pitch, heave and flap motion:

$$\mathbf{u} = \begin{bmatrix} \alpha \\ \dot{\alpha} \\ h \\ \dot{h} \\ \delta \\ \dot{\delta} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \rho_{1,1} \\ \rho u_{1,1} \\ \rho v_{1,1} \\ e_{1,1} \\ \vdots \\ \rho_{imax,jmax} \\ \rho u_{imax,jmax} \\ \rho v_{imax,jmax} \\ e_{imax,jmax} \end{bmatrix} \mathbf{y} = \begin{bmatrix} C_l \\ C_d \\ C_m \\ C_h \end{bmatrix} \quad (3)$$

When constructing a reduced order model we generate a linearised form of (1) and (2):

$$\frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (4)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad (5)$$

where the system matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ can be identified from impulse responses of the CFD scheme via system identification. Popular methods include proper orthogonal decomposition (POD) [15] and the eigenvalue realisation algorithm (ERA) [3,16]. Typically an impulse response or a series of frequency responses is required to each input of $\mathbf{u}$ (3). In this work we shall consider the impulse responses as used in [3].

In practice a pulse input magnitude of less than unity is required to prevent breakdown and poorly converged solutions of the CFD model. In the following section we will discuss a procedure by which the pulse response can estimated prior to running the unsteady CFD.

3 NUMERICAL PROCEDURE.

Classical one dimensional piston theory describes the unsteady pressure at a point on a moving surface as being analogous to a piston moving through a one dimensional channel [17–19] . Using Bernoulli's equation and isentropic relations the pressure on the face of a one dimensional piston is:

$$\frac{[p_1]_i}{[p_0]_i} = \left(1 + \frac{(\gamma - 1)}{2} \frac{[v_1]_i}{[a_0]_i}\right)^{\frac{2\gamma}{(\gamma-1)}} \quad (6)$$

The subscripts 0 and 1 refer to the states before and after the perturbation respectively. Here we have adopted the method of Zhang et al. [20] where the piston theory is applied around the local pressure at each i^{th} cell, rather than the freestream conditions. p is the local static pressure, a is the local speed of sound and γ is the adiabatic index (assumed here to be 1.4) and v is the wall normal surface fluid velocity due to the perturbation. A third order binomial expansion of Eqn.(6) is given by [17, 18] :

$$[p_1]_i - [p_0]_i \quad = \quad [\rho_0]_i \left[a_0^2\right]_i \left[\frac{[v_1]_i}{[a_0]_i} + \frac{\gamma(\gamma+1)}{4} \left(\frac{[v_1]_i}{[a_0]_i} \right)^2 + \frac{\gamma(\gamma+1)}{12} \left(\frac{[v_1]_i}{[a_0]_i} \right)^3 \right] \tag{7}$$

The wall normal surface fluid velocity v_1 can be described in terms of the surface normals as:

$$v_1 = \Delta\mathbf{V} \cdot \hat{\mathbf{n}}_1 + \mathbf{V} \cdot (\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_0) \tag{8}$$

where $\hat{\mathbf{n}}$ is a unit normal vector on the body surface. $\mathbf{V}$ is the unperturbed surface fluid velocity vector and $\Delta\mathbf{V}$ is the change in the surface fluid velocity vector due to the aerofoil motion. In our analysis ΔV is the prescribed surface velocity.

3.1 Time step limitation

One dimensional piston theory has been shown to give good results so long as any of the following is true [17, 21]:

$$M^2 \gg 1 \tag{9}$$

$$\kappa M^2 \gg 1 \tag{10}$$

$$\kappa^2 M^2 \gg 1 \tag{11}$$

where M is the freestream mach number and κ is the reduced frequency:

$$\kappa = \frac{\omega c}{U_\infty} \tag{12}$$

ω is the circular frequency, c is the chord and U_∞ is the freestream velocity. For a pulse response the reduced frequency is better expressed as:

$$\kappa = \frac{c}{\Delta t \cdot U_\infty} \tag{13}$$

If the freestream velocity and chord are nondimensionalised such that $c/U = \mathcal{O}(1)$, then:

$$\kappa = \frac{1}{\Delta t} \tag{14}$$

hence the condition:

$$\kappa^2 M^2 = \left[\frac{M}{\Delta t} \right]^2 \gg 1 \tag{15}$$

This is the condition that shall be investigated in the following chapters.

4 RESULTS

To test the accuracy of the pressure on the first time step directly after a pulse, a number of Euler simulations are compared with the piston analysis presented in section 3.

The results shown here are for a NACA 0012 aerofoil with a 25% flap and pitching about the quarter chord on a course mesh with 139x15 cells (Fig. 1). The flow solver is an implicit cell centered finite volume dual time Euler scheme [12, 13, 22] . Equation (8) for the pulse inputs $\mathbf{u}$ (3) can be expressed as:

$$v_1 = \begin{cases} \mathbf{V} \cdot (\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_0) & \alpha \text{ Pitch pulse} \\ (x - a_x)\,\dot{\alpha} \cdot \hat{\mathbf{n}}_1 & \dot{\alpha} \text{ Pitch rate pulse} \\ 0 & h \text{ Heave pulse} \\ \dot{\mathbf{h}} \cdot \hat{\mathbf{n}}_1 & \dot{h} \text{ Heave rate pulse} \\ \mathbf{V} \cdot (\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_0) & \delta \text{ Flap Pulse} \\ (x - a_x)\,\dot{\alpha} \cdot \hat{\mathbf{n}}_1 & \dot{\delta} \text{ Flap rate pulse} \end{cases} \tag{16}$$

The normal velocity component for a pure displacement in heave is predicted to be zero. Here the aerofoil does not undergo any rotation, and all surface velocities due to the displacement are zero. We will observe in the following sections, that the nonlinear response to a pure heave displacement, although nonzero, remains very small.

Tables 1 and 2 give an overview of the test cases used. The nonlinear mean flow solutions are given in Fig. 2. The unsteady pulse responses (Fig. 3-10) are shown as a change in pressure and integral force from the nonlinear mean solution, where:

$$\Delta p = p_1 - p_0 \tag{17}$$

$$\Delta C_F = [C_F]_1 - [C_F]_0 \tag{18}$$

where p is the nondimensional static pressure and C_F is the respective integral force coefficient ($F = L, M, H$ for lift, pitching moment and hinge moment). Again, subscripts 0 and 1 represent the values before after the first time step when the pulse is applied. All four pulses have been superimposed against a rescaled axes (with respect to its scaling factor *sca* shown in Table 1) to demonstrate the approximate linearity of the response.

Test cases 1 and 2 show where the piston theory approximation works well i.e. $\kappa^2 M^2$ is sufficiently large. Test cases 3 and 4 show a flow where the criteria for $\kappa^2 M^2$ is violated. The largest pulse size (sca=125) is selected to be unfavourably large. Many of the responses for the large pulse inputs are seen to be highly nonlinear, for test case 4 responses to pitch (α) and heave rate ($\dot{h}$) introduced shock waves which were not present in the steady flow. In general these large pulse inputs struggled with convergence and were accompanied by very large changes in the integral force values. For test cases 1 to 3 the initial integral values for lift and pitching moment are accurately predicted by the piston theory. As can be seen from the pressure responses, the piston theory cannot capture the merging of the trailing edge pressure with the wake and hence hinge moment coefficients are less accurate to predict. Test case 4 is run at an unrealistically high time step which would not normally be encountered for aeroelastic simulations. However even here (case 4) the correct order of magnitude of the integral forces and the maximum/minimum surface pressures is captured by the piston theory.

Pulse	$[units]$	$sca = 1$	sca =5	sca =25	sca =125
α	$[°]$	0.08	0.4	2	10
$\dot{\alpha}$	$[°/s]$	0.4	2	10	50
h	$[c]$	0.001	0.005	0.025	0.125
$\dot{h}$	$[c/s]$	0.008	0.04	0.2	1.0
δ	$[°]$	0.08	0.4	2	10
$\dot{\delta}$	$[°/s]$	0.4	2	10	50

Table 1: Pulse inputs

Case	α_0	Ma	Δt_{REAL}	$\kappa^2 M^2$
1	0.0	0.7	0.2	12.25
2	0.0	0.8	0.2	16
3	2.0	0.3	0.2	2.25
4	0.0	0.7	2.0	0.1225

Table 2: Test Cases

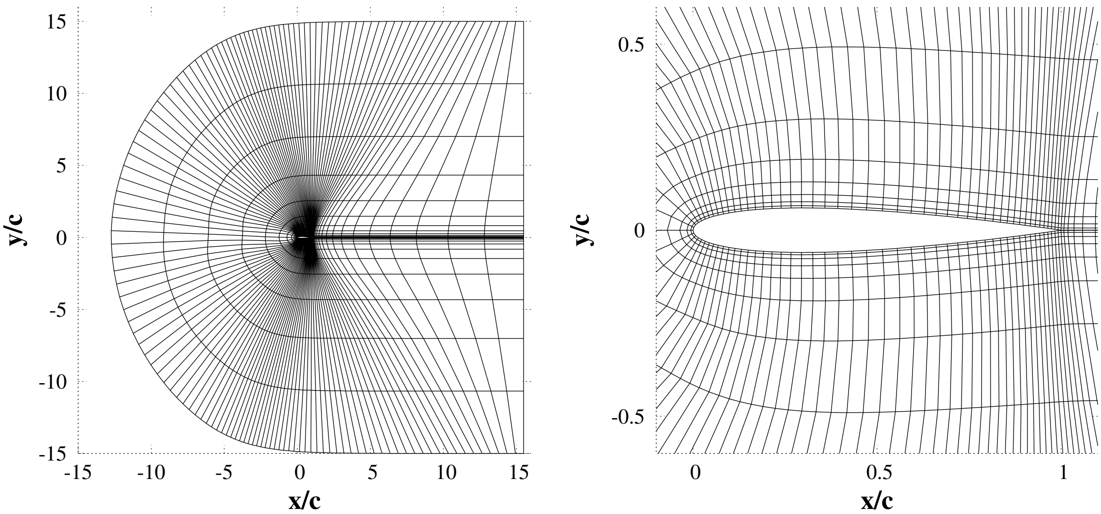

Figure 1: Finite volume Euler mesh
(139x15 cells, 99 cells over the aerofoil and a first cell height of 0.006c)

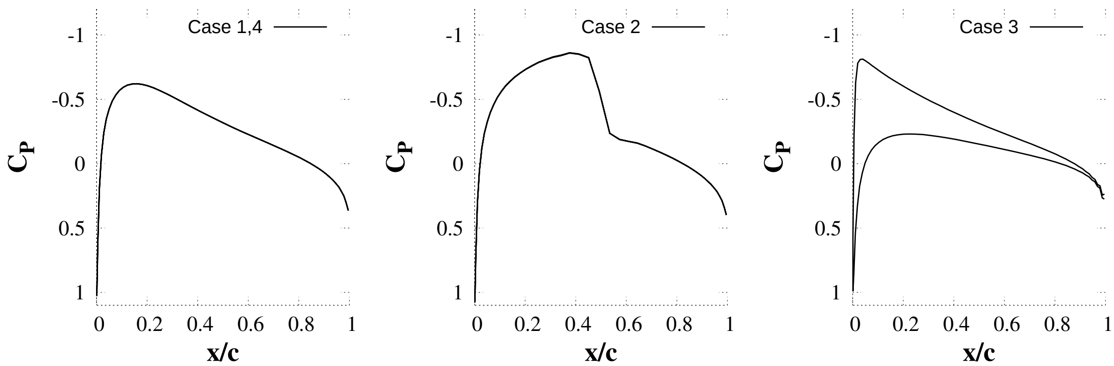

Figure 2: Nonlinear mean flow solutions

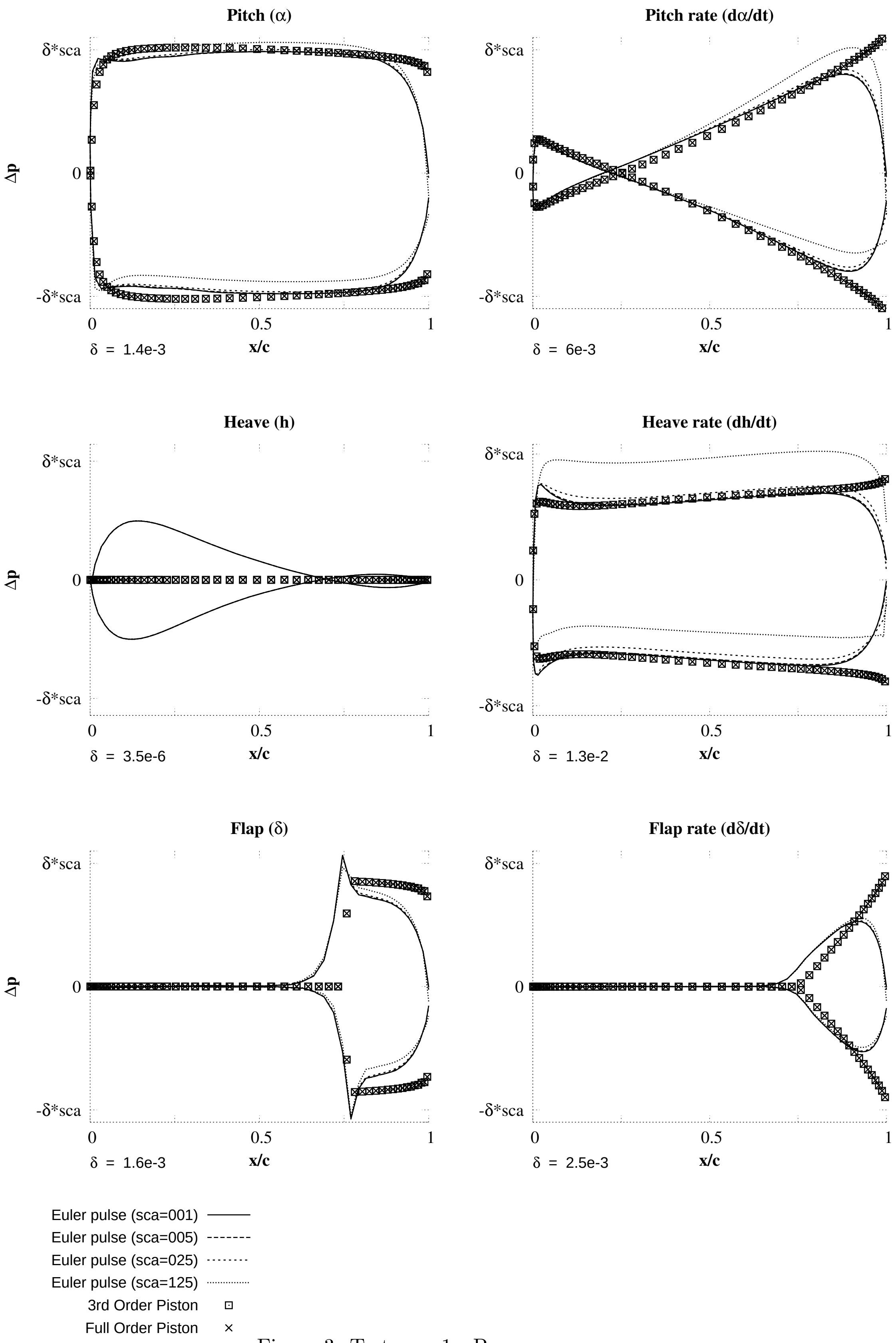

Figure 3: Test case 1 - Pressure response

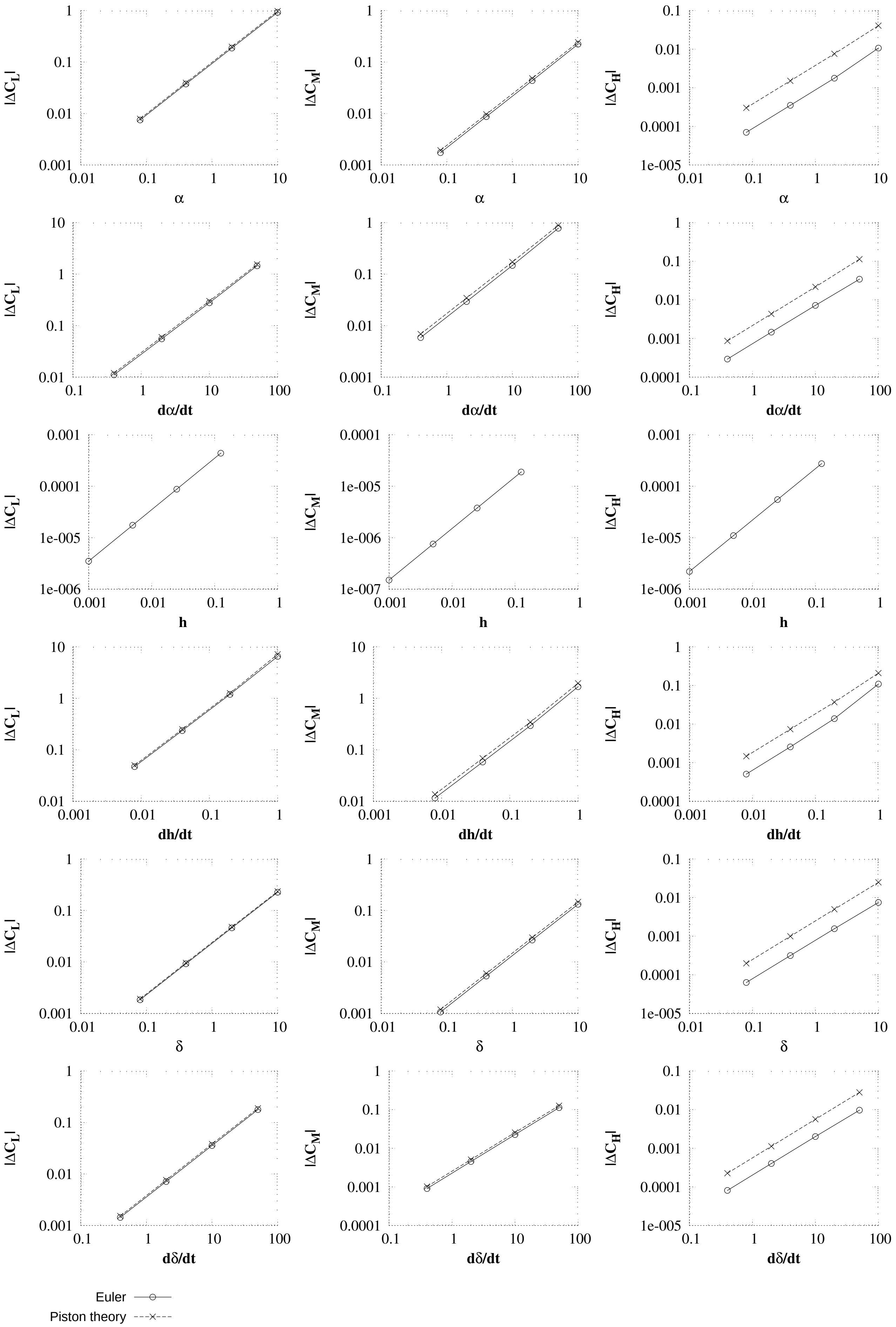

Figure 4: Test case 1 - Integral forces

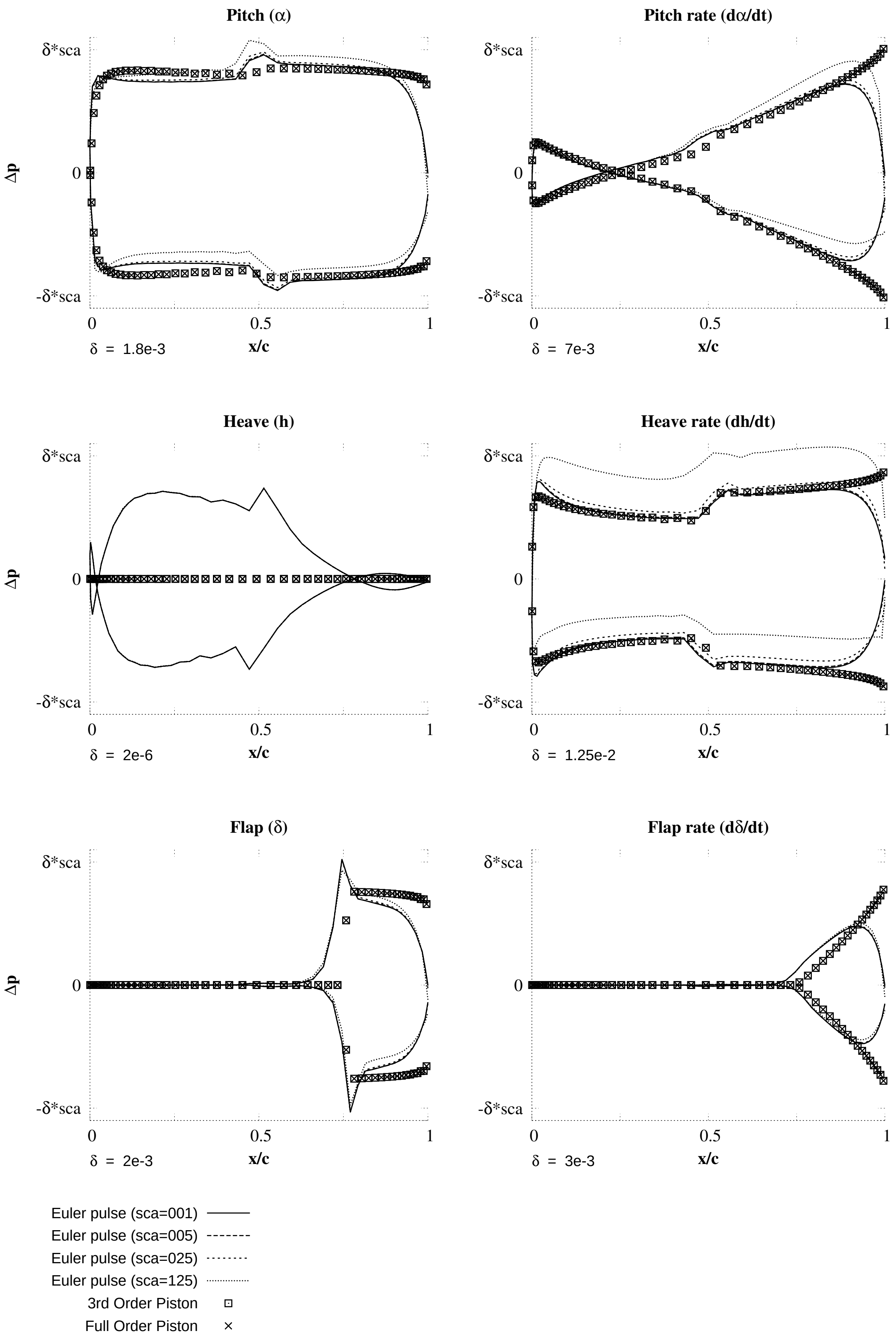

Figure 5: Test case 2 - Pressure response

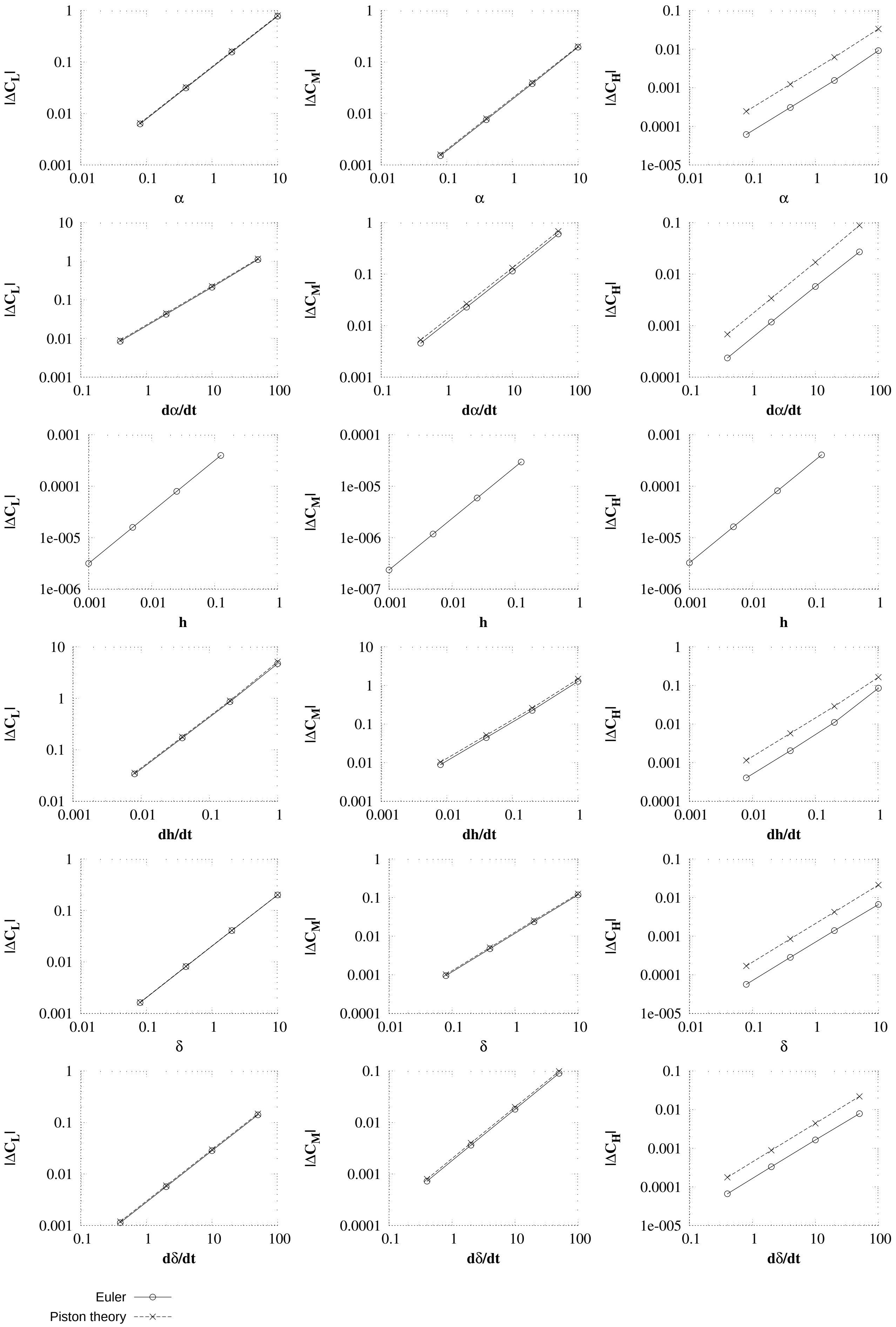

Figure 6: Case 2 - Integral forces

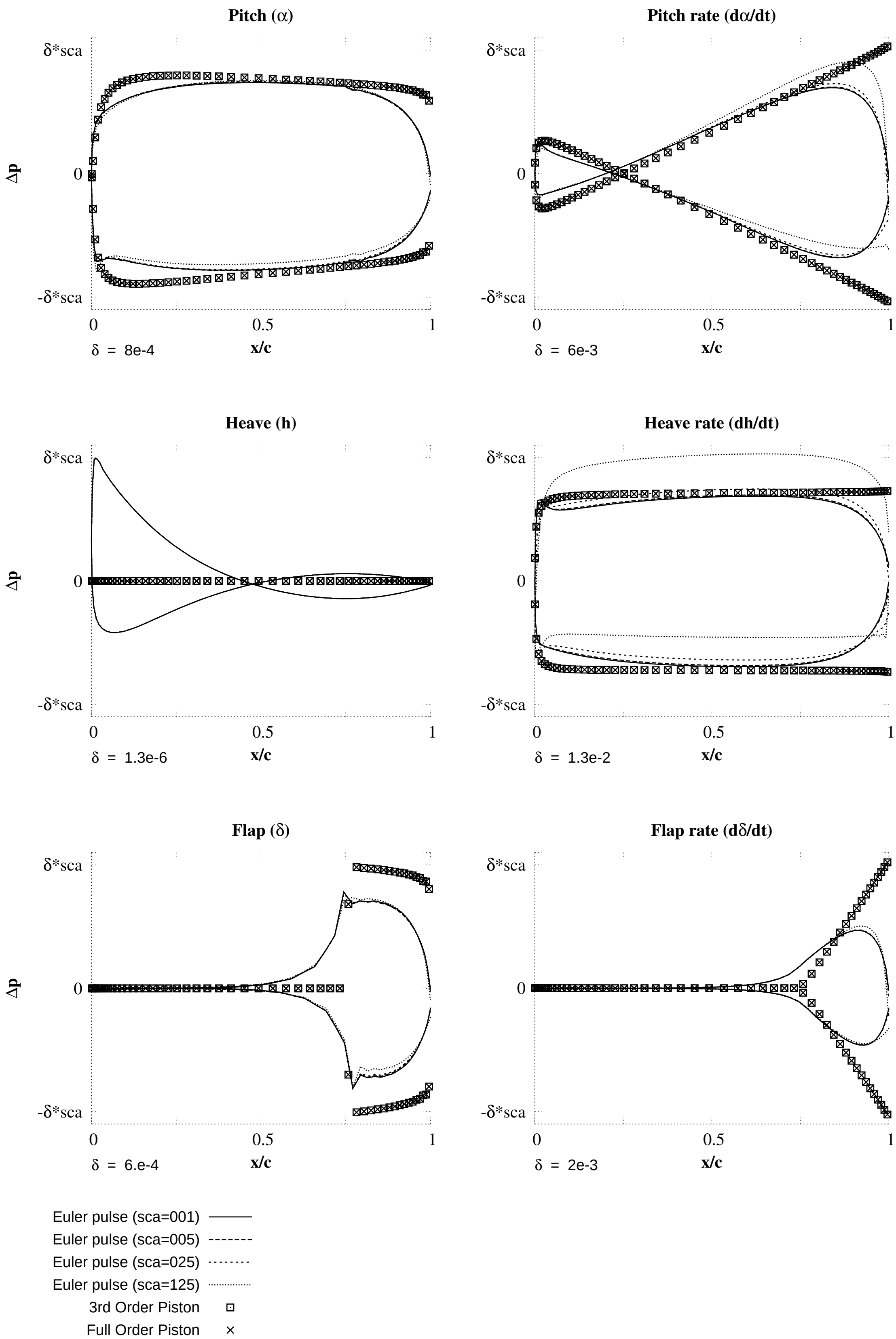

Figure 7: Test case 3 - Pressure response

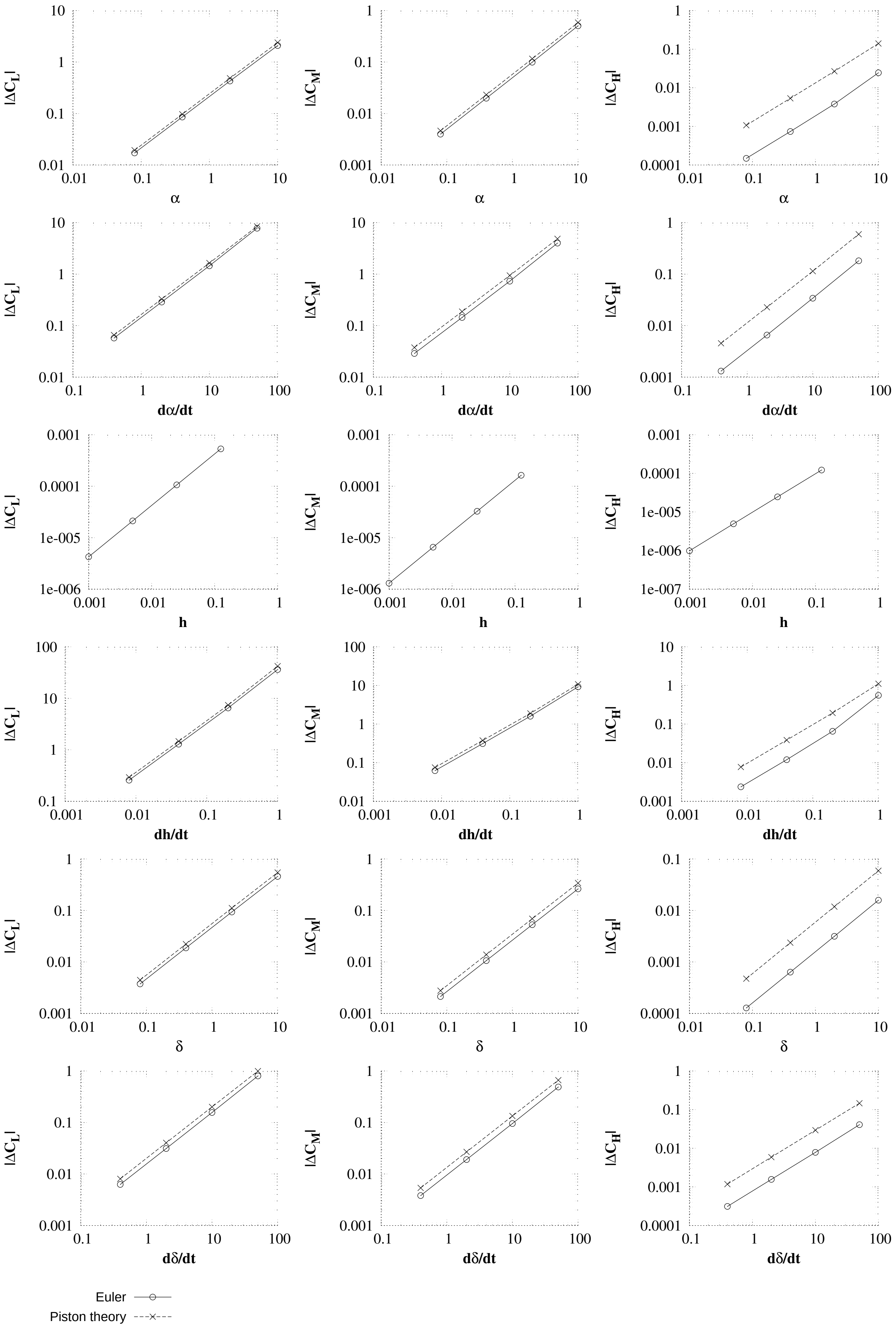

Figure 8: Test case 3 - Integral forces

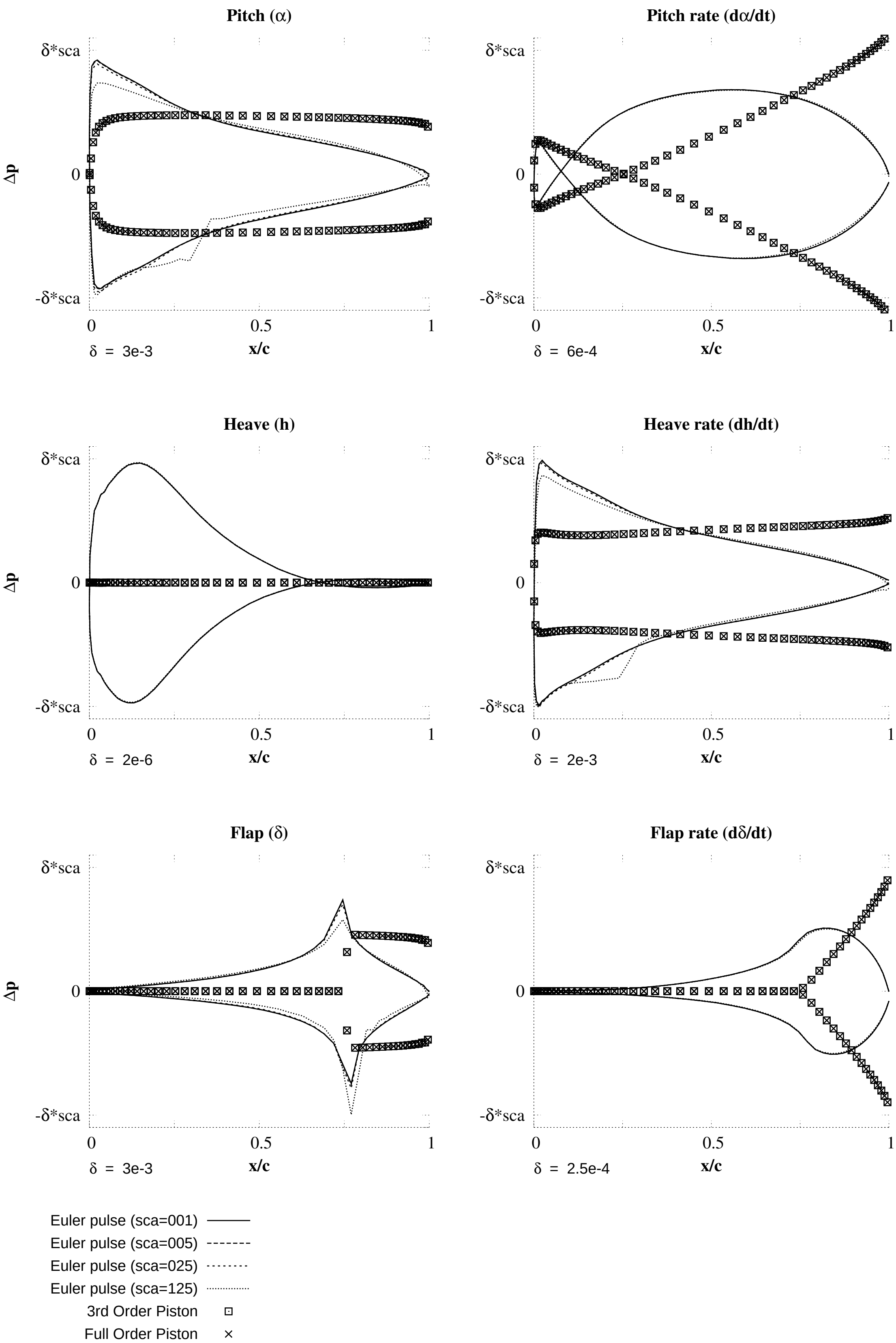

Figure 9: Test case 4 - Pressure response

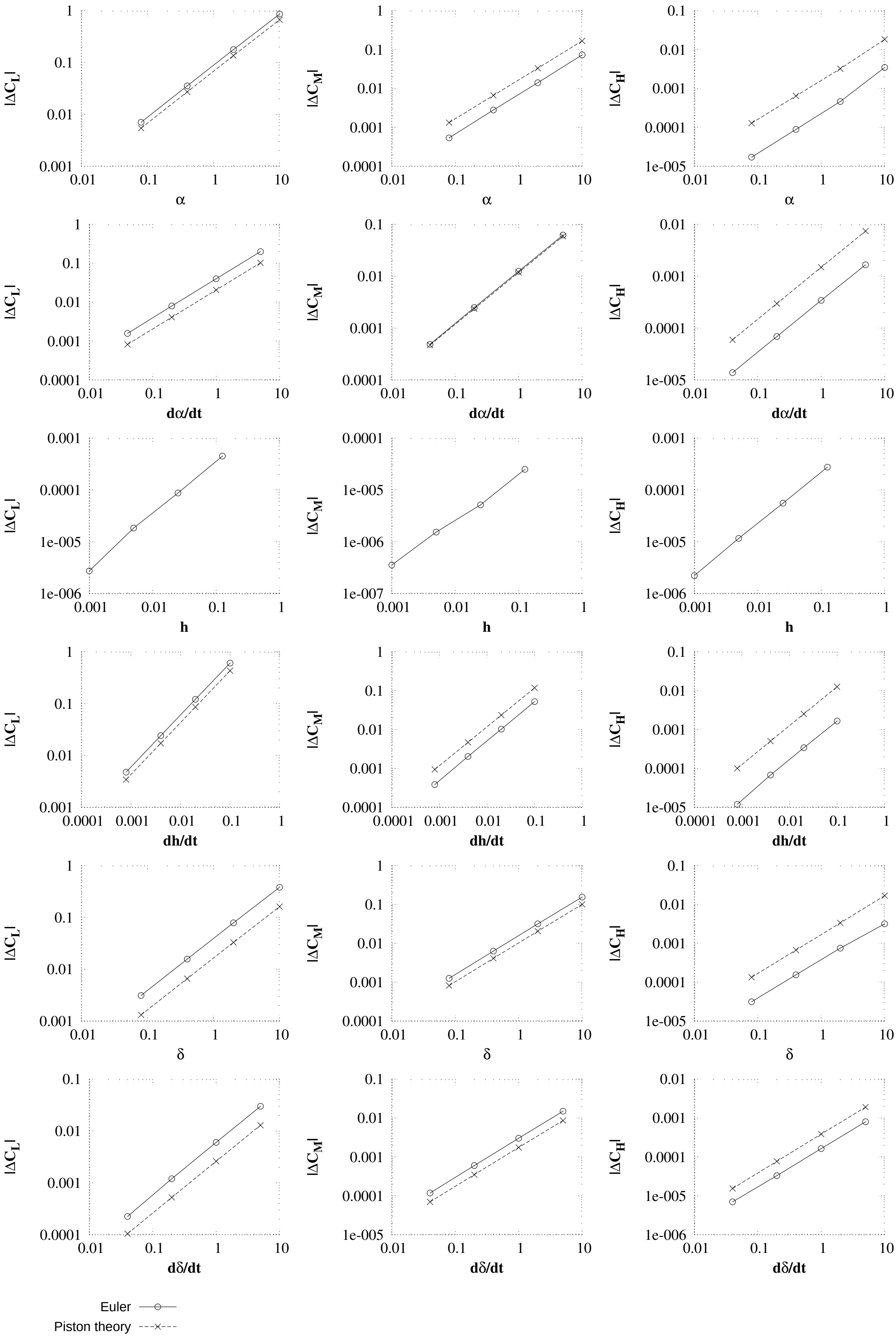

Figure 10: Test case 4 - Integral forces

5 PULSE SIZING

If the pulse is too large, the Euler equations can reach non-physical solutions. The most critical cases to avoid are supersonic velocities moving away from the aerofoil surface. Under these conditions, the pressure on the surface can approach zero, and in a nonphysical case the unconverged transient pressure may become negative. A simple limitation on the pressure to be greater than zero was futile for large pulses, as even here the low pressures giving rise to very high Mach numbers persist to retard the convergence of the solution.

A further limiting factor is the CFD response to the forced input dropping below the accuracy of the CFD scheme (i.e. the residual limit as specified by the user). The decay of a free response is exponential so care should be taken with very small pulse inputs.

Prescribing a required change in the integral forces is generally enough to ensure a sensible response. Limits should however still be applied on maximum displacements to ensure mesh integrity is maintained. A safety check on minimum pressure (or its corresponding wall normal Mach number) is encouraged. For most cases the author usually prescribes[1] $\Delta C_L = 0.01 \ldots 0.1$ depending on the steady state pressure distribution. In the first instance, heave responses are set to approximately $h = \mathcal{O}\left(0.01c\right)$ and ensuring that the CFD is converged down to a low residual such that the response is captured.

To use piston theory as a tool for pulse sizing, the following work flow is applied:

1. Generate nonlinear mean flow solution
2. Apply local one-dimensional piston theory (6) to find the constrained pulse input magnitude for the Euler equations.
3. Apply the pulse input to the Euler equations.
4. Check the final response is as expected and fully captured within the accuracy of the converged solution.

6 REFERENCES

[1] Balajewicz, M. (2010). Reduced order modeling of transonic flutter and limit cycle oscillations using the pruned volterra series. In *51st AIAA/ASME/ASCE/AHS/ASC Structure, Structural Dynamics, and Materials Conference. AIAA-2010-2725.*

[2] Dowell, E. H., Hall, K. C., Thomas, J. P., et al. (1999). Reduced order models in unsteady aerodynamics. In *40th AIAA/ASME/ASCE/AHS/ASC Structure, Structural Dynamics, and Materials Conference. AIAA-99-1261.*

[3] Gaitonde, A. L. and Jones, D. P. (2006). Study of linear response identification techniques and reduced-order model generation for a 2D CFD scheme. *International Journal for Numerical Methods in Fluids*, 52(12), 1361–1402. ISSN 02712091.

[4] Grewal, A. and Zimcik, D. (2008). Application of Reduced Order Modelling to Fluid-Structure Interaction Analysis. In *AIAA/ISSMO Multidisiplinary Analysis and Optimisation Conference AIAA-2008-5961.*

[1] Intended only as a helpful guideline and chosen only as it often produces a suitable response on a limited number of test cases. Convergence speed due to the pulse input has not been considered.

[5] Kim, T., Hong, M., Bhatia, K., et al. (2005). Aeroelastic Model Reduction for Affordable Computational Fluid Dynamics-Based Flutter Analysis. *AIAA Journal*, 43(12), 2487–2495.

[6] Marzocca, P., Librescu, L., and Silva, W. (2001). Volterra series approach for nonlinear aeroelastic response of 2-D lifting surfaces. In *42nd AIAA/ASME/ASCE/AHS/ASC Structure, Structural Dynamics, and Materials Conference. AIAA-2001-1459.*

[7] Morino, H., Yamaguchi, H., Kumano, T., et al. (2009). Efficient Aeroelastic Analysis Using Unstructured CFD Method and Reduced-Order Unsteady Aerodynamic Model. In *50th AIAA/ASME/ASCE/AHS/ASC Structure, Structural Dynamics, and Materials Conference. AIAA-2009-2326.*

[8] Raveh, D. and Mavris, D. (2001). Reduced-order models based on CFD impulse and step responses. In *42nd AIAA/ASME/ASCE/AHS/ASC Structure, Structural Dynamics, and Materials Conference. AIAA-01-1527.*

[9] Sarkar, S. (2004). Model order reduction of unsteady flow past oscillating airfoil cascades. *Journal of Fluids and Structures*, 19(2), 239–247. ISSN 08899746.

[10] Silva, W. (1999). Reduced-order models based on linear and nonlinear aerodynamic impulse responses. In *40th AIAA/ASME/ASCE/AHS/ASC Structure, Structural Dynamics, and Materials Conference. AIAA-99-1262.*

[11] Silva, W. and Bartels, R. (2004). Development of reduced-order models for aeroelastic analysis and flutter prediction using the CFL3Dv6.0 code. *Journal of Fluids and Structures*, 19, 729–745. ISSN 08899746.

[12] Jameson, A., Schmidt, W., and Turkel, E. (1981). Numerical solution of the euler equations by finite volume mehtodfs using runge-kutta time-stepping schemes. In *AIAA 14th Fluid and Plasma Dynamic Conference. AIAA-1981-1259.*

[13] Kroll, N. and Jain, R. K. (1987). *Solution of Two-Dimensional Euler Equations - Experience with a finite volume code.* Forschungsbericht, DFVLR-FB 87-41, Braunschweig, Germany.

[14] Gaitonde, A. L. (1994). A dual time method for the solution of the unsteady euler equations. *Aeronautical Journal*, 98, 283–291.

[15] Hall, K. C., Dowell, E. H., and Thomas, J. P. (2000). Proper orthogonal decomposition technique for transonic unsteady aerodynamic flows. *AIAA Journal*, Vol. 38 issue 10, 1853–1862.

[16] J.-N, Juang, and Pappa, R. S. (1985). An eigensystem realization algorithm for modal parameter identification and model reduction. *Journal of Guidance, Control, and Dynamics*, Vol 8. No. 5, 620–627.

[17] Ashley, H. and Zartarian, G. (1956). Piston theory - a new aerodynamic tool for the aeroelastician. *Journal of the aeronautical sciences*, 23, 1109–1118.

[18] Lighthill, M. J. (1953). Oscillating airfoils at high mach number. *Journal of the Aeronautical sciences*, Vol. 20(6), 402–406.

[19] Van Dyke, M. D. (1953). Supersonic flow past oscillating airfoils including nonlinear thickness effects. Tech. rep., NACA technical note 2982.

[20] Zhang, W., Ye, Z., C., Z., et al. (2009). Supersonic Flutter Analysis Based on a Local Piston Theory. *AIAA Journal*, 47(10), 2321–2328. ISSN 0001-1452. doi: 10.2514/1.37750.

[21] Landahl, M., Mollo-Christensen, E. L., and Ashley, H. (1955). Parametric studies of viscous and vonviscous unsteady flows. Tech. rep., O.S.R. Technical Report 55-13.

[22] Jameson, A. (1984). A non-oscillatory shock capturing scheme using flux limited dissipation. In *Large-scale computations in fluid mechanics; Proceedings of the Fifteenth Summer Seminar on Applied Mathematics*.